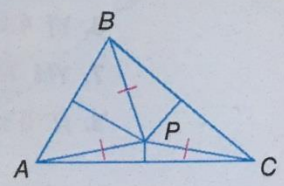
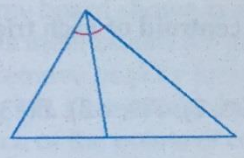
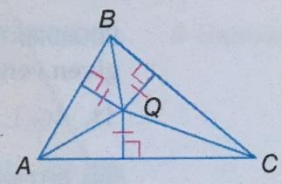
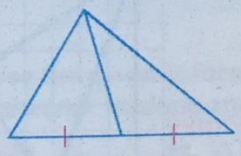
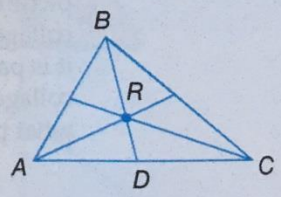
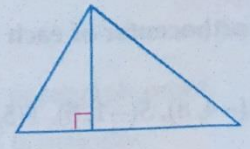
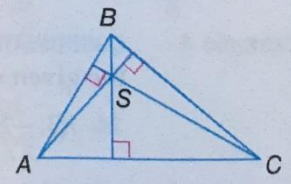
# Summary Sheet from 7-1 to 7-2

**ConceptSummary Special Segments and Points in Triangles**

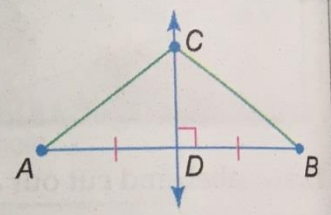
| Name | Example | Point of Concurrency | Special Property | Example |
|---|---|---|---|---|
| perpendicular bisector |  | circumcenter | The circumcenter $P$ of $\triangle ABC$ is equidistant from each vertex. |   |
| angle bisector |  | incenter | The incenter $Q$ of $\triangle ABC$ is equidistant from each side of the triangle. |   |
| median |  | centroid | The centroid $R$ of $\triangle ABC$ is two thirds of the distance from each vertex to the midpoint of the opposite side. |   |
| altitude |  | orthocenter | The lines containing the altitudes of $\triangle ABC$ are concurrent at the orthocenter $S$. |   |

## Theorems Perpendicular Bisectors

**7.1 Perpendicular Bisector Theorem**

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

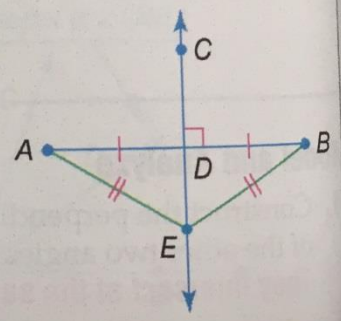
**Example:** If $\overline{CD}$ is a $\perp$ bisector of $\overline{AB}$, then $AC = BC$.




**7.2 Converse of the Perpendicular Bisector Theorem**

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

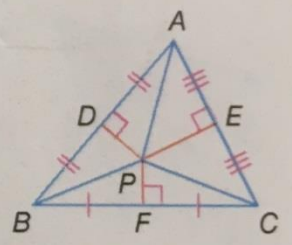
**Example:** If $AE = BE$, then $E$ lies on $\overline{CD}$, the $\perp$ bisector of $\overline{AB}$.




## Theorem 7.3 Circumcenter Theorem

**Words** The perpendicular bisectors of a triangle intersect at a point called the *circumcenter* that is equidistant from the vertices of the triangle.

**Example** If $P$ is the circumcenter of $\triangle ABC$, then $PB = PA = PC$.

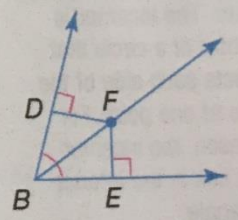



## Theorems Angle Bisectors

**7.4 Angle Bisector Theorem**

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

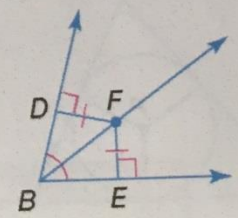
**Example:** If $\overrightarrow{BF}$ bisects $\angle DBE$, $\overline{FD} \perp \overrightarrow{BD}$, and $\overline{FE} \perp \overrightarrow{BE}$, then $DF = FE$.




**7.5 Converse of the Angle Bisector Theorem**

If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the bisector of the angle.

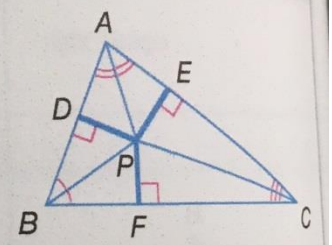
**Example:** If $\overline{FD} \perp \overrightarrow{BD}$, $\overline{FE} \perp \overrightarrow{BE}$, and $DF = FE$, then $\overrightarrow{BF}$ bisects $\angle DBE$.



**Theorem 7.6 Incenter Theorem**

**Words** The angle bisectors of a triangle intersect at a point called the *incenter* that is equidistant from the sides of the triangle.

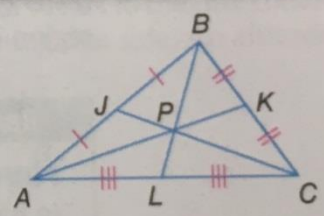
**Example** If $P$ is the incenter of $\triangle ABC$, then $PD = PE = PF$.




**Theorem 7.7 Centroid Theorem**

The medians of a triangle intersect at a point called the centroid that is two thirds of the distance from each vertex to the midpoint of the opposite side.

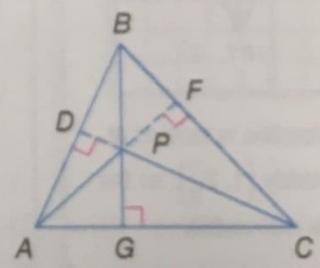
**Example** If $P$ is the centroid of $\triangle ABC$, then $AP = \frac{2}{3}AK$, $BP = \frac{2}{3}BL$, and $CP = \frac{2}{3}CJ$.




**KeyConcept Orthocenter**

The lines containing the altitudes of a triangle are concurrent, intersecting at a point called the orthocenter.

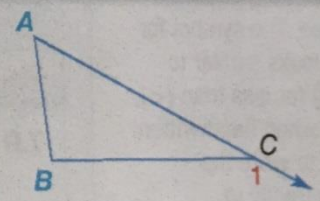
**Example** The lines containing altitudes $\overline{AF}$, $\overline{CD}$, and $\overline{BG}$ intersect at $P$, the orthocenter of $\triangle ABC$.



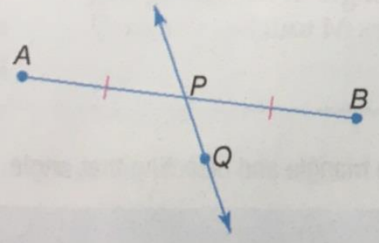

**Theorem 7.8 Exterior Angle Inequality**

The measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles.

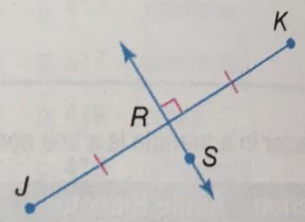
**Example:** $m\angle 1 > m\angle A$
$m\angle 1 > m\angle B$



**1 Perpendicular Bisectors** A segment bisector is any segment, line, or plane that intersects a segment at its midpoint. If a bisector is also perpendicular to the segment, it is called a **perpendicular bisector**.



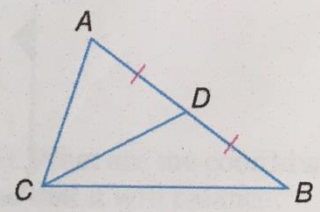

$\overleftrightarrow{PQ}$ is a bisector of $\overline{AB}$.




$\overleftrightarrow{RS}$ is a perpendicular bisector of $\overline{JK}$.

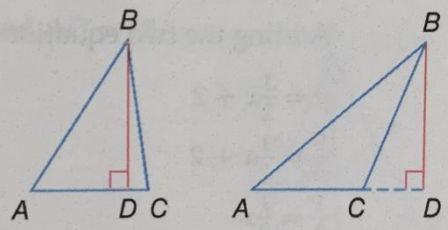
**1 Medians** A **median** of a triangle is a segment with endpoints being a vertex of a triangle and the midpoint of the opposite side.

Every triangle has three medians that are concurrent. The point of concurrency of the medians of a triangle is called the **centroid** and is always inside the triangle.




$\overline{CD}$ is a median of $\triangle ABC$.

**2 Altitudes** An **altitude** of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side. An altitude can lie in the interior, exterior, or on the side of a triangle.




$\overline{BD}$ is an altitude from $B$ to $\overline{AC}$.

Every triangle has three altitudes. If extended, the altitudes of a triangle intersect in a common point.